1　式の計算

A

1

(1)

$$\begin{aligned}与式&=\{(x-2)(x+1)\}\{(x-4)(x+3)\}+24\\&=\{(x^2-x)-2\}\{(x^2-x)-12\}+24\\&=(x^2-x)^2-14(x^2-x)+48\\&=\{(x^2-x)-6\}\{(x^2-x)-8\}\\&=(x^2-x-6)(x^2-x-8)\\&=(x+2)(x-3)(x^2-x-8)\end{aligned}$$

(2)

$$\begin{aligned}与式&=(y+z+3)x+y^2+yz+5y+2z+6\\&=(y+z+3)x+(y+2)z+y^2+5y+6\\&=(y+z+3)x+(y+2)z+(y+2)(y+3)\\&=(y+z+3)x+(y+2)\{z+(y+3)\}\\&=(y+z+3)x+(y+2)(y+z+3)\\&=\{x+(y+2)\}(y+z+3)\\&=(x+y+2)(y+z+3)\end{aligned}$$

補足

次数の小さい文字式順に整理すると楽。つまり，与式はxの 1 次式，zの 1 次式，yの 2 次式だから，まずxまたはzについて整理

上のようにxについて整理した場合は，次に，yとzの式の部分をzについて整理，そして，yの 2 次式を因数分解

(3)

$$\begin{aligned}与式&=(a^2-2ab+b^2)c+(b^2-2bc+c^2)a+(c^2-2ca+a^2)b+8abc\\&=(b+c)a^2+(b^2+2bc+c^2)a+bc(b+c)\\&=(b+c)a^2+(b+c)^2a+bc(b+c)\\&=(b+c)\{a^2+(b+c)a+bc\}\\&=(b+c)\{(a+b)(a+c)\}\\&=(a+b)(b+c)(c+a)\end{aligned}$$

(4)

$$\begin{aligned}与式&=\{(b+c)a+b(b+c)\}(c+a)+abc\\&=(b+c)a^2+\{b(b+c)+c(b+c)\}a+bc(b+c)+abc\\&=(b+c)a^2+\{(b+c)^2+bc\}a+bc(b+c)\\&=\{a+(b+c)\}\{(b+c)a+bc\}\\&=(a+b+c)(ab+bc+ca)\end{aligned}$$

(5)

$$
\begin{aligned}
与式 &= a^4 - 2(b^2 + c^2)a^2 + b^4 - 2b^2c^2 + c^4 \\
&= a^4 - 2(b^2 + c^2)a^2 + (b^2 - c^2)^2 \\
&= \{a^2 - (b^2 + c^2)\}^2 - (b^2 + c^2)^2 + (b^2 - c^2)^2 \\
&= (a^2 - b^2 - c^2)^2 - 4b^2c^2 \\
&= (a^2 - b^2 - c^2)^2 - (2bc)^2 \\
&= \{(a^2 - b^2 - c^2) - 2bc\}\{(a^2 - b^2 - c^2) + 2bc\} \\
&= \{a^2 - (b^2 + 2bc + c^2)\}\{a^2 - (b^2 - 2bc + c^2)\} \\
&= \{a^2 - (b + c)^2\}\{a^2 - (b - c)^2\} \\
&= \{a + (b + c)\}\{a - (b + c)\}\{a + (b - c)\}\{a - (b - c)\} \\
&= (a + b + c)(a - b - c)(a + b - c)(a - b + c)
\end{aligned}
$$

2

(1)

$$
\begin{aligned}
与式 &= \frac{2}{(1+\sqrt{2})+\sqrt{3}} \cdot \frac{(1+\sqrt{2})-\sqrt{3}}{(1+\sqrt{2})-\sqrt{3}} + \sqrt{\frac{4-2\sqrt{3}}{2}} \\
&= \frac{2(1+\sqrt{2}-\sqrt{3})}{(1+\sqrt{2})^2-(\sqrt{3})^2} + \frac{\sqrt{(\sqrt{3}-1)^2}}{\sqrt{2}} \\
&= \frac{2(1+\sqrt{2}-\sqrt{3})}{2\sqrt{2}} + \frac{\sqrt{3}-1}{\sqrt{2}} \\
&= \frac{1+\sqrt{2}-\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{3}-1}{\sqrt{2}} \\
&= 1
\end{aligned}
$$

(2)

$$
\begin{aligned}
与式 &= \frac{(2-\sqrt{5}i)(1-\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)} + \frac{(2+\sqrt{5}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} \\
&= \frac{(2-\sqrt{15})-(2\sqrt{3}+\sqrt{5})i}{1+3} + \frac{(2-\sqrt{15})+(2\sqrt{3}+\sqrt{5})i}{1+3} \\
&= \frac{2-\sqrt{15}}{2}
\end{aligned}
$$

よって，実部は $\frac{2-\sqrt{15}}{2}$，虚部は 0

3

(1)

$$\text{与式}=\frac{1}{1-\dfrac{1}{\dfrac{-x}{1-x}}}$$
$$=\frac{1}{1+\dfrac{1-x}{x}}$$
$$=\frac{1}{\dfrac{1}{x}}$$
$$=x$$

(2)

$$\text{与式}=1+\frac{2}{x}-\frac{(x+1)+2}{x+1}-\frac{(x-3)-2}{x-3}+\frac{(x-4)-2}{x-4}$$
$$=1+\frac{2}{x}-\left(1+\frac{2}{x+1}\right)-\left(1-\frac{2}{x-3}\right)+1-\frac{2}{x-4}$$
$$=2\left(\frac{1}{x}-\frac{1}{x+1}+\frac{1}{x-3}-\frac{1}{x-4}\right)$$
$$=2\left\{\frac{(x+1)-x}{x(x+1)}+\frac{(x-4)-(x-3)}{(x-3)(x-4)}\right\}$$
$$=2\left\{\frac{1}{x(x+1)}-\frac{1}{(x-3)(x-4)}\right\}$$
$$=2\cdot\frac{-8x+12}{x(x+1)(x-3)(x-4)}$$
$$=-\frac{8(2x-3)}{x(x+1)(x-3)(x-4)}$$

補足

分子の式の次数を分母の式の次数より小さくすることから始める。

4

$\omega^2+\omega+1=0$ より，$\omega(\omega^2+\omega+1)=0$

また，$\omega^2+\omega=-1$ だから，

$$\omega(\omega^2+\omega+1)=\omega^3+\omega^2+\omega$$
$$=\omega^3-1$$

よって，$\omega^3-1=0$ すなわち $\omega^3=1$

これより，

$$
\begin{aligned}
\text{与式} &= (a+b+c)\{a+b\omega+c(-\omega-1)\}\{a+b(-\omega-1)+c\omega^3\cdot\omega\} \\
&= (a+b+c)\{(a-c)+(b-c)\omega\}\{a-b(\omega+1)+c\omega\} \\
&= (a+b+c)\{(a-c)+(b-c)\omega\}\{(a-b)-(b-c)\omega\} \\
&= (a+b+c)\left[(a-c)(a-b)+\{(a-b)-(a-c)\}(b-c)\omega-(b-c)^2\omega^2\right] \\
&= (a+b+c)\left\{(a-c)(a-b)-(b-c)^2\omega-(b-c)^2\omega^2\right\} \\
&= (a+b+c)\left\{(a-c)(a-b)-(b-c)^2(\omega^2+\omega)\right\} \\
&= (a+b+c)\left\{(a-c)(a-b)-(b-c)^2\cdot(-1)\right\} \\
&= (a+b+c)\left\{(a-c)(a-b)+(b-c)^2\right\} \\
&= (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \\
&= a^3+b^3+c^3-3abc
\end{aligned}
$$

5

(1)

$$
\begin{aligned}
\text{与式} &= (x^4+1)^2-2x^4 \\
&= (x^4+1)^2-(\sqrt{2}x^2)^2 \\
&= \{(x^4+1)+\sqrt{2}x^2\}\{(x^4+1)-\sqrt{2}x^2\} \\
&= \left\{(x^2+1)^2-(2-\sqrt{2})x^2\right\}\left\{(x^2+1)^2-(2+\sqrt{2})x^2\right\} \\
&= \left\{(x^2+1)^2-\left(\sqrt{2-\sqrt{2}}x\right)^2\right\}\left\{(x^2+1)^2-\left(\sqrt{2+\sqrt{2}}x\right)^2\right\} \\
&= \left\{(x^2+1)+\sqrt{2-\sqrt{2}}x\right\}\left\{\left(x^2+1-\sqrt{2-\sqrt{2}}x\right)\right\}\left\{(x^2+1)+\sqrt{2+\sqrt{2}}x\right\}\left\{(x^2+1)-\sqrt{2+\sqrt{2}}x\right\} \\
&= \left(x^2+\sqrt{2-\sqrt{2}}x+1\right)\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)
\end{aligned}
$$

(2)

$$
\begin{aligned}
\text{与式} &= (a+b)^3-3ab(a+b)-3ab+1 \\
&= (a+b)^3+1^3-3ab\{(a+b)+1\} \\
&= \{(a+b)+1\}^3-3(a+b)\cdot1\{(a+b)+1\}-3ab(a+b+1) \\
&= (a+b+1)^3-3(a+b)(a+b+1)-3ab(a+b+1) \\
&= (a+b+1)\left\{(a+b+1)^2-3(a+b)-3ab\right\} \\
&= (a+b+1)(a^2+b^2-ab-a-b+1)
\end{aligned}
$$

補足： $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

(3)

$$
\begin{aligned}
\text{与式} &= (x-y)^3+(z-y)^3-\{(x-y)+(z-y)\}^3 \\
&= (x-y)^3+(z-y)^3-\left\{(x-y)^3+(z-y)^3+3(x-y)(z-y)\{(x-y)+(z-y)\}\right\} \\
&= 3(x-y)(z-y)(x-2y+z)
\end{aligned}
$$

6

(1)

$$与式=\left[\left(9+4\sqrt{5}\right)^n+\left(9-4\sqrt{5}\right)^n+\left\{\left(9+4\sqrt{5}\right)^n-\left(9-4\sqrt{5}\right)^n\right\}\right]$$
$$\times\left[\left(9+4\sqrt{5}\right)^n+\left(9-4\sqrt{5}\right)^n-\left\{\left(9+4\sqrt{5}\right)^n-\left(9-4\sqrt{5}\right)^n\right\}\right]$$
$$=4\left(9+4\sqrt{5}\right)^n\left(9-4\sqrt{5}\right)^n=4\left\{\left(9+4\sqrt{5}\right)\left(9-4\sqrt{5}\right)\right\}^n=4\cdot 1^n=4$$

(2)

$$与式=\left\{\left(\frac{1+i}{\sqrt{2}}\right)^2\right\}^p+\left\{\left(\frac{1-i}{\sqrt{2}}\right)^2\right\}^p=i^p+(-i)^p=i^p+(-1)^p i^p$$

p は奇数だから，$(-1)^p=-1$　$\therefore$ 与式$=i^p-i^p=0$

(3)

$$与式=-\frac{a^3}{(a-b)(c-a)}-\frac{b^3}{(a-b)(b-c)}-\frac{c^3}{(c-a)(b-c)}$$
$$=-\frac{a^3(b-c)+b^3(c-a)+c^3(a-b)}{(a-b)(b-c)(c-a)}$$
$$=-\frac{(b-c)a^3-\left(b^3-c^3\right)a+b^3c-bc^3}{(a-b)(b-c)(c-a)}$$
$$=-\frac{(b-c)a^3-(b-c)\left(b^2+bc+c^2\right)a+bc(b+c)(b-c)}{(a-b)(b-c)(c-a)}$$
$$=-\frac{(b-c)\left\{a^3-\left(b^2+bc+c^2\right)a+bc(b+c)\right\}}{(a-b)(b-c)(c-a)}$$
$$=-\frac{(b-c)\left(a^3-ab^2-abc-ac^2+b^2c+bc^2\right)}{(a-b)(b-c)(c-a)}$$
$$=-\frac{(b-c)\left\{(c-a)b^2+\left(c^2-ca\right)b+a^3-ac^2\right\}}{(a-b)(b-c)(c-a)}$$
$$=-\frac{(b-c)\left\{(c-a)b^2+c(c-a)b-a(c-a)(c+a)\right\}}{(a-b)(b-c)(c-a)}$$
$$=-\frac{(b-c)(c-a)\left\{b^2+bc-a(c+a)\right\}}{(a-b)(b-c)(c-a)}$$
$$=-\frac{(b-c)(c-a)\left\{-(a-b)c-\left(a^2-b^2\right)\right\}}{(a-b)(b-c)(c-a)}$$
$$=\frac{(b-c)(c-a)\left\{(a-b)c+(a-b)(a+b)\right\}}{(a-b)(b-c)(c-a)}$$
$$=\frac{(b-c)(c-a)(a-b)(c+a+b)}{(a-b)(b-c)(c-a)}$$
$$=a+b+c$$

7

$$\left(3+2\sqrt{2}\right)^4-\left\{1-\left(3-2\sqrt{2}\right)^4\right\}=\left(3+2\sqrt{2}\right)^4+\left(3-2\sqrt{2}\right)^4-1$$
$$=\left\{\left(3+2\sqrt{2}\right)^2+\left(3-2\sqrt{2}\right)^2\right\}^2-2\left(3+2\sqrt{2}\right)^2\left(3-2\sqrt{2}\right)^2-1$$
$$=34^2-2\left\{\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)\right\}^2-1$$
$$=1156-2\cdot 1-1$$
$$=1153$$

より，

$$\left(3+2\sqrt{2}\right)^4=1153+\left\{1-\left(3-2\sqrt{2}\right)^4\right\} \quad \cdots ①$$

$2^2<8<3^2$ より， $2<2\sqrt{2}<3$

したがって， $-3<-2\sqrt{2}<-2$

これより， $3+(-3)<3+\left(-2\sqrt{2}\right)<3+(-2)$ 　すなわち　$0<3-2\sqrt{2}<1$

よって， $0<\left(3-2\sqrt{2}\right)^4<1$

ゆえに， $0<1-\left(3-2\sqrt{2}\right)^4<1 \quad \cdots ②$

①，②より， $\left(3+2\sqrt{2}\right)^4$ の小数部分は $1-\left(3-2\sqrt{2}\right)^4$ である。